# Chapter 28: Input-output and other matrix-based analyses 

## A. Introduction

28.1 The purpose of this chapter is to build on the presentation of the supply and use tables in chapter 14 to examine in greater detail the possibilities offered by using a matrix form of presentation of the accounts. As has been noted on a number of occasions, the SNA is intended to offer a degree of flexibility in implementation as long as the inherent accounting rules are observed. The fact that the requirement to balance uses and resources is immediately obvious within a matrix framework makes this a powerful way in which to explore different options while still ensuring the balances are satisfied. One aim of this chapter is to demonstrate the power of a matrix presentation in this way.

## 1. Input-output tables

28.2 A second aim is to describe the basic ideas of input-output matrices. Supply and use tables are an integral part of the SNA and the process of compiling these tables is a powerful way of ensuring consistency between the various data sources available to the compiler. For many analytical purposes, though, a transformation from a pair of supply and use tables into a single input-output table where row and column totals are equal brings very considerable advantages. Input-output tables cannot be compiled without passing through the supply and use stage (except under very restrictive assumptions). They are therefore analytical constructs that inevitably involve some degree of modelling in their compilation.
28.3 There is a vast literature on the compilation and use of input-output tables and it is impossible in a short chapter to give a full appreciation of the range of complexities of compilation and inventiveness of applications. The chapter aims only to give a feel for the sort of operations necessary to transform supply and use tables into input-output tables and to give some ideas of their possible applications. The Manual of Supply, Use and Input-Output Tables and a visit to the web site of the International Input-Output Association (www.iioa.org) are good places to start a more detailed investigation of the potential in this field.

## 2. Social accounting matrices

28.4 Both the supply and use tables and input-output tables are matrix representations of the goods and services account. It
is possible to cast the whole of the sequence of accounts, including the goods and services account, in a matrix format also. Such a matrix is called a social accounting matrix (SAM).
28.5 It is possible to extend and elaborate a SAM by introducing alternative disaggregations of existing flows or new types of flows, just as long as the use and resource of these flows balance in the usual way. This is such a common extension of a SAM that the usual understanding of what a SAM is often goes further than a matrix encompassing the standard sequence of accounts to include extensions, particularly of the household sector.

## 3. The structure of the chapter

28.6 Chapter 14 describes how the supply and use tables may be used in order to ensure the internal consistency of disparate data sets. Section B of this chapter looks at two particular aspects of the supply and use tables where it may be useful to adopt a different approach to that described in chapter 14. The first of these concerns the treatment of insurance and freight on imported goods and the second concerns the treatment of goods that are processed by a unit that is not the legal owner of them. Section B also discusses how information cross-classified by establishment and industry can be transformed into information relating to institutional sectors.
28.7 Section C is concerned with how a pair of supply and use tables may be transformed into a single symmetric inputoutput matrix. Each of the supply and use tables shows disaggregation by products and industries. In an inputoutput table, one of these dimensions is eliminated. Thus a single table may show the relationship between the supply and use of products or alternatively the output of industries and the demand for the output of industries.
28.8 Section D goes on to show how the whole of the accounting system can be represented in matrix form. This is a useful pedagogical tool and may be instructive as a stepping-off point for extensions of the accounts such as social accounting matrices.

## B. Flexibility in the supply and use tables

## 1. The treatment of margins on imports

28.9 In discussing valuation in section B of chapter 14, consideration is given to how transport margins should be incorporated into the accounts and in particular how international transport charges should be recorded. Paragraphs 14.61 to 14.77 explain that the parallel between basic and producer prices does not carry forward simply to a distinction between CIF and FOB-based prices. The distinction depends on whether it is the unit providing the goods or the unit taking delivery of the goods that is responsible for providing the transport and insurance. Paragraph 14.77 ends by discussing briefly the practical problems in deriving the desired valuation from the available data sources. It is reproduced here for convenience.
28.10 It may not be possible to determine from customs declarations which unit is responsible for the transport costs and, even when it is and conceptually the transport costs should be separated from the value of the goods themselves, there may be no information and no resources available to make the separation in practice. In such a case the CIF value of imports may be the only source with a disaggregation by type of good. If the disaggregated CIF figures are used for imports of goods, though, that part of the transport costs and insurance also included in imports of services would be double-counted. In order to avoid this, therefore, an adjustment column is inserted into the supply table. The adjustment column consists of a deduction from the services items for transport and insurance equal to the CIF-to-FOB adjustment for these items with an offsetting global adjustment made to imports of goods. Table 14.4, reproduced here as table 28.1 gives an example of such an adjustment.

Table 28.1:An example of imports entries in the supply table with the global CIF to FOB adjustment

|  | CIF/FOB adjustment | Goods | Services |
| :---: | :---: | :---: | :---: |
| (0) | 37 |  |  |
| Ores and minerals; electricity, gas and water (1) | 61 |  |  |
| Manufacturing (2-4) | 284 |  |  |
| Construction (5) |  |  |  |
| Trade, accommodation, food \& beverages transport services (6) | -6 62 |  |  |
| Finance and Insurance (7 less 72-73) | -4 17 |  |  |
| Real estate services; and rental and leasing services (72-73) |  |  |  |
| Business and production services (8) $\quad 5$ |  |  |  |
| Community and social services (92-93) |  |  |  |
| Other services (94-99) |  |  |  |
| Public administration (91) |  |  |  |
| CIF/FOB adjustment | 10 | -10 |  |
| Purchases abroad by residents |  | 20 | 23 |
| Total | 0 | 392 | 107 |

28.11 This adjustment column shows the reallocation of service margins from the industries where they are produced (by resident or non-resident producers) to an adjustment row for the CIF/FOB adjustment. In the column for goods, the
values given industry by industry include an element of these service margins, but this is deducted on the CIF/FOB adjustment row to leave the total equal to the total of imports FOB. The adjustments in this column are analogous to a similar column that could be shown illustrating the adjustment between purchasers' and basic prices.
28.12 A simpler procedure than that just described, though one not strictly consistent with BPM6 recommendations, is to ignore the balance of payments division between goods and services and adjust the figures for imports of services by the amount of services provided by non-residents that are included in the detailed figures for imports of goods. This ensures that the total of imports of goods and services agrees with the total in the balance of payments but will not agree with the total of imports of goods FOB and of services shown there. This makes compiling the supply and use tables simpler but means that it is not possible to use imports of goods on a FOB basis to match exports of those goods from other countries. Even in this simpler version, however, the amount of freight and insurance on imports provided by residents must be shown as an export of services.

## 2. Goods processed by a unit not assuming economic ownership

28.13 A producer may carry out the same activity under quite different economic conditions. Consider farmers growing grain which is milled into flour before use. Suppose one farmer acquires a mill to process his own grain but once this is acquired he may offer to mill grain for others for a fee. The production account for the farmer with a mill will look somewhat different from that for a farmer who does not have a mill but pays the first farmer a fee for milling even though both produce flour for sale.
28.14 In the case of milling the reasons for subcontracting the activity to another may be the availability of suitable fixed capital. Increasingly, however, similar processes are being carried out internationally and in respect of activities more usually associated with manufacturing such as the assembling of component parts. Here the motivation is less one of the availability of capital than of the costs of labour. If the average wages in country X are half of those in country Y, it may be cost-effective for a unit in Y to dispatch the components to a unit in X for assembly and then have the completed product returned to Y or even shipped directly to a final purchaser.
28.15 Previous editions of the SNA have recommended that components for assembly should be recorded as delivered to the unit in country X and that the whole of the value of the completed product should be recorded as output of X and exports from X to Y . This does not match the treatment of grain milling or, for example, repairs to machinery where no such change of ownership of the goods being processed is imputed. Imputing a change of ownership of the parts to be assembled gives rise to significant data compilation problems because the value of the assembled product may be greater than the cost of the components
plus the fee to assemble them. The value of the finished product may incorporate the results of research and development of the unit contracting the assembly, for instance. The SNA now recommends that products should only be recorded as being delivered to another unit if there is a change of ownership or, in the case where both producing units belong to the same enterprise, the producing unit taking delivery also assumes responsibility for subsequent risks and rewards of production such as deciding how much to process, what price to charge and when to sell.
28.16 The question arises of how to record the activity of assembling goods to order for another unit in the supply and use tables and the input-output table. The processes of assembly for oneself and for another are physically similar but the economics are different.
28.17 Suppose in year 1 a processing unit converts products only on own account. In year 2 the unit processes the same amount on its own account but also processes a similar amount on behalf of another. Suppose the cost of items processed in year 1 is 90 , the cost of associated products needed to assemble them is 10 and the value added is 35 . The total value of output is thus 135. In year 2, all other things being equal, intermediate consumption increases by another 10 to 110 and value added to 70 bringing the value of output to 180 . The change in the structure of production is difficult to understand in the absence of information on the change in the role of the producer who is operating no longer only on his own behalf but also on behalf of others.
28.18 There are essentially two ways to proceed. The first is to treat processing on own account and on behalf of another as different types of activity and different products. In this way in the second year the producer would have one activity with inputs of 100 , value added of 35 and output of 135 as in the first year, plus another activity with inputs of 10 , value added 35 and output of 45 .

## Table 28.2: Options for recording goods not changing economic ownership

|  | Year 1 | Year 2 | Option 1 | Option 2 |  |
| :--- | ---: | ---: | :---: | ---: | ---: |
| Cost of materials | 90 | 90 | 90 |  | 180 |
| Other costs | 10 | 20 | 10 | 10 | 20 |
| Total intermediate |  |  |  |  |  |
| consumption | 100 | 110 | 100 | 10 | 200 |
| Value added | 35 | 70 | 35 | 35 | 70 |
| Output | 135 | 180 | 135 | 45 | 270 |

28.19 The second alternative is to show the intermediate inputs in the second year as 200 , value added as 70 and output as 270. Value added is the same under both options and the comparison between the second and the first year makes more sense from a transformation point of view under option 2. However, adding an extra 90 to both output and intermediate consumption is essentially artificial. Further, as noted above, it may be difficult for the processor to put a value on the components he receives and the output he provides to the other unit. The chances are that he only knows that he receives a fee of 45 to cover his incidental
expenses of 10 and leave an amount of value added, 35 in this case. These options are shown in table 28.2.
28.20 It should be emphasized that it is option 1 that is the recommendation of the SNA and, for goods sent abroad for processing, BPM6. Option 2 is shown as a supplementary presentation that may be adopted for reasons of continuity with past practices. Option 1 more accurately reflects the economic processes taking place while option 2 focuses on the physical transformation process.
28.21 When goods are sent abroad for processing, they are recorded as neither exports of goods by the country holding economic ownership, nor as imports of goods by the processing country in either the SNA or BPM6. Similarly, after processing they are recorded neither as exports by the processing country nor as imports of goods by the country of economic ownership. The only item recorded as imports and exports is the fee agreed between the economic owner and the processor.
28.22 The physical flows of the goods will continue to appear in the merchandise trade figures. However, the product code after processing may be different from the code on entry, making it difficult to match the incoming and outgoing flows.
28.23 The presentation of option 2 suggests that the fee can be derived as the difference between the value of the goods on arrival and departure from the processing country but while this may sometimes give a reasonable approximation of the processing fee, there are many reasons why this may not be so.
a. If processing takes any significant amount of time, there may be holding gains and losses affecting the value of the goods. These accrue to the economic owner, not the processor.
b. Goods may be lost or damaged or may simply become obsolete while in process. (This has been observed in the case of electronic components.) These other volume changes also apply to the economic owner and not the processor.
c. The value of the processed goods may be greater than the costs of the components and the processing fee to the extent that the finished product incorporates part of the value of R\&D treated as fixed capital formation of the economic owner.
28.24 All these situations reinforce the preference for option 1 over option 2 in table 28.2.

## 3. Supply and use tables and sector accounts

28.25 As explained in chapter 14 , it is possible to derive the three estimates of GDP from a set of supply and use tables. Since these tables can be expressed in volume terms, estimates can also be made of growth rates based on the tables. However, to complete the sequence of accounts, production accounts are needed by institutional sector. To ensure that the supply and use table and the sequence of accounts are perfectly integrated and consistent, it is desirable to take the

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part of the use table showing intermediate consumption and the components of value added and allocate the columns to institutional sectors.
28.26 The starting point for the compilation is the part of the use table in table 14.12 relating to intermediate consumption and value added. This is shown in a somewhat aggregated form in table 28.3.
28.27 The easiest allocation is for financial corporations since typically such corporations do not undertake secondary activity and other institutional units do not undertake any financial activity. When these conditions prevail, the column for the finance and insurance activity can be taken in its entirety as appropriate for the institutional sector. It is possible that financial corporations may undertake some production for own final use (as capital formation), in which case some part of an appropriate column in the section of table 28.3 relating to own account production should be added. No such adjustment has been made in this example.
28.28 The columns relating to non-market producers must be allocated between general government and NPISHs. In addition, though not in this example, it is possible that either general government or NPISHs may have an
establishment undertaking market production. This is how it is possible that non-market producers may have small amounts of operating surplus. It is also possible that both general government and NPISHs may have some production for own final use (as capital formation) but none has been assumed here.
28.29 The last step is to allocate all columns not yet accounted for between non-financial corporations and households. An indication that some part of a market production activity should be allocated to households is the presence of mixed income as part of the value added of the activity. Thus, in this example, some parts of market production of agriculture, manufacturing, construction and trade are attributable to households as well as production for own final use. (As noted in general some of production for own final use will be attributable to other sectors. It is not done so here for reasons of simplicity at such an aggregate level.)
28.30 Once these calculations are complete, table 28.4 results, showing for each sector not just total intermediate consumption but also a product breakdown of this as well as the items for value added.
28.31 The figures shown for intermediate consumption, output and the elements of value added for each institutional

Table 28.3:The use table from table 14.12

sector are those that appear in the production account and generation of income account in the sequence of accounts.

## C. Deriving an input-output table

## 1. What is an input-output table?

28.32 Essentially an input-output table is derived from a use table where either the columns representing industries in the two left-most quadrants are replaced by products or where the products in the two topmost quadrants are replaced by industries. The resulting intermediate consumption matrix is then square, showing products in both rows and columns or industries in both. In both cases the row totals for the complete matrix match the column totals for the complete matrix, product by product or industry by industry as the case may be. The resulting matrices are therefore referred to as being symmetric.
28.33 The process of replacing the product dimension by an industry one is based on one of several possible models, to be discussed below. This process necessarily means that a symmetric input-output matrix is further removed from basic data sources than a supply and use table and it is therefore useful to review why making this transition is so useful.
28.34 Note that in table 14.12, there is a product for ores and minerals, electricity and water but no column for it. If there is no industry for which this is the principal product, identifying the primary producers rather than the number of products will determine the final size of the symmetric (square) matrix.

Table 28.3 (cont):The use table from table 14.12


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2. Analytical potential of an input-output matrix
28.35 Such tables have algebraic properties that make them particularly suitable for analyses that enable estimates to be made of the effects of changing relative prices, of labour and capital requirements in the face of changing output levels, of the consequences of changing patterns of demand and so on. They may also may be used as the basis for an expanded version that may be used to estimate the demands made by the economy on the environment, for instance.
28.36 As noted in the introduction, there is a vast literature on how to compile and use input-output tables. The purpose of this section is simply to indicate the key aspects of converting a pair of supply and use tables into an inputoutput table.
28.37 Suppose the entries in the inter-industry matrix are each divided by the figure for output at the bottom of the corresponding column, and the resulting matrix is designated as A; the vector of outputs is written as x and the vector of total final demand is written as $y$. Then
$A x+y=x$
This can be rewritten as
$(I-A) x=y$
or
$x=(I-A)^{-1} y$.
28.38 The matrix (I-A) is known as the Leontief matrix, after the man who pioneered the use of input-output tables and the matrix $(\mathrm{I}-\mathrm{A})^{-1}$ is known as the Leontief inverse. It is the last formulation that gives the analytical power to input-output analysis.
28.39 Suppose there is an increase in demand, for manufactured products, say. Looking at even the supply and use table it can be seen that to increase the output of these goods, more inputs of almost all types of products are needed. This increase in demand for a range of products is called the direct effect of a change in demand. However, the increase in demand in all these products causes a further round of increases in output for all products and this in turn triggers another set of increases in output and so on. Each round of effects is smaller than the last until it eventually becomes insignificant. The total of all second and subsequent round effects is called the indirect effect of a change in demand.

Table 28.4:Intermediate consumption and value added cross-classified by industry and institutional sector

28.40 In terms of the algebra just introduced, the direct effect is equal to $A y$, the second round effect to $A^{2} y$, the third round effect to $\mathrm{A}^{3} \mathrm{y}$ and so on. It can be shown that (I-A) ${ }^{-1}$ can be written as $\mathrm{A}+\mathrm{A}^{2}+\mathrm{A}^{3}+\mathrm{A}^{4}$ etc. This is where the power of having a symmetric matrix comes from since A needs to be square for this formulation to work.
28.41 As long as changes in demand, y , are sufficiently small that the average coefficients in A are likely to be good approximations to the new situation, the new level of x can be calculated. The approach breaks down if the changes in demand are so great that significant changes in A are likely to follow and marginal rather than average coefficients are needed.
28.42 The matrix A is also sometimes called a matrix of technological coefficients and can provide insights into the way an economy works. In an economy dominated by primary products with little processing carried out in the domestic economy, there are relatively few significant nonzero elements in A. As the economy develops and processing of primary products becomes more commonplace, A becomes more populated with entries reflecting greater vertical and horizontal integration of activities within the economy. By exploring different industries associated with different stages in the production process it is possible to say where value added is generated.

For example, cotton is grown as an agricultural product. It is then subject to separation into lint and seed (ginning), then the lint is converted to yarn and the yarn to fabric. If each of these activities appears in a different industry, it is possible to see where the value added between the growing of the cotton and the eventual fabric in which it is used arises.

## 3. Secondary products

28.43 An industry classification such as ISIC essentially identifies industries in terms of the sorts of goods or services they typically produce. However, there are more products than industries and, for all sorts of reasons, some products may be made in several industries.
28.44 In order to limit the number of products per unit and to allow integration with basic production statistics, the concept of establishment is introduced. In principle, an establishment produces only one product at one location but the SNA recognizes that in practice it is not possible to separate production into such fine detail. Dealing with the fact that many establishments produce more than one product is fundamental to the idea of calculating a symmetric input-output matrix.

Table 28.4 (cont):Intermediate consumption and value added cross-classified by industry and institutional sector

28.45 The reason that manipulation of supply and use tables is needed to produce an input-output table is the existence of secondary products. If there were the same number of industries as products, and if each industry only produced one product, the supply table for the domestic economy would be unnecessary; the column totals for industries would be numerically equal to the row totals for products and the inter-industry matrix would be square as originally compiled. As noted elsewhere, the intent behind using establishments rather than enterprises, and working at a fairly detailed level in the supply and use tables, is to get as close to this situation as is reasonably practicable. Inevitably though some secondary production remains.
28.46 There are three types of secondary production
a. Subsidiary products: those that are technologically unrelated to the primary product. Just a few examples include a large retailer with a fleet of trucks used primarily for its own purposes that may occasionally offer transport services to another unit, a farmer who use part of his land as a caravan site, or a mining company that builds access roads and accommodation for its workers.
b. By-products: products that are produced simultaneously with another product but which can be regarded as secondary to that product, for example gas produced by blast furnaces.
c. Joint products: products that are produced simultaneously with another product that cannot be said to be secondary (for example beef and hides).

In order to reduce the supply and use tables to one single input-output matrix two possibilities exist. One is to express the input-output matrix in terms of products only;
the other is to express the input-output table in terms of industries.

## 4. Reallocating secondary products

28.47 There are two basic approaches to eliminating secondary products. Both come from applying information from the use matrix to the supply matrix to reduce it to a purely diagonal one. Once this is done, the supply matrix contains no further useful information and is no longer presented. The transformed use matrix is what is referred to as an input-output matrix.
28.48 In deriving a product by product matrix in the simplest possible way, the final demand quadrant of the use matrix is unaltered. It already expresses demand by product and does not need changing. The intermediate consumption and value added parts of the matrix, though, need to be changed from an industry dimension to a product one. The row totals of the matrix already show the correct product totals so the exercise consists of reallocating entries from one column to another within the given row total. This is called a technology approach. It assumes that the demand for intermediate consumption and labour and capital inputs are determined by the nature of the products made.
28.49 In deriving an industry by industry matrix in the simplest possible way, the value added part of the use matrix is unaltered and because the level of output will not alter, only the composition of intermediate consumption changes, not its total. Thus the exercise is one of reallocating items between rows but not between columns. In contrast to the product by product case, the quadrant relating to final demand will change and will show demand related to the industry supplying the products and not to the products themselves. This is called a sales structure approach. It assumes that as the level of output of an industry changes, the pattern of sales will remain the same.

Table 28.5:A numerical example of reallocating products from construction to manufacturing

| Use of products |  |  |  |  | Industry technology |  | Product technology |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |
| 1 Agriculture, forestry and fishery products (0) | 71 | 0 | 3.8 | 0.0 | 71.0 | 0.0 | 71.2 | -0.2 |
| 2 Ores and minerals; electricity, gas and water (1) | 190 | 1 | 10.2 | 0.5 | 190.0 | 1.0 | 190.6 | 0.4 |
| 3 Manufacturing (2-4) | 675 | 63 | 36.3 | 30.3 | 676.8 | 61.2 | 677.2 | 60.8 |
| 4 Construction (5) | 9 | 5 | 0.5 | 2.4 | 9.1 | 4.9 | 9.0 | 5.0 |
| 5 Trade, accommodation, food \& beverages; transport services (6) | 65 | 3 | 3.5 | 1.4 | 65.1 | 2.9 | 65.2 | 2.8 |
| 6 Finance and Insurance (7 less 72-73) | 36 | 5 | 1.9 | 2.4 | 36.1 | 4.9 | 36.1 | 4.9 |
| 7 Real estate services; and rental and leasing services (72-73) | 15 | 1 | 0.8 | 0.5 | 15.0 | 1.0 | 15.0 | 1.0 |
| 8 Business and production services (8) | 70 | 12 | 3.8 | 5.8 | 70.3 | 11.7 | 70.2 | 11.8 |
| 9 Community and social services (92-93) | 1 | 0 | 0.1 | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 10 Other services (94-99) | 1 | 0 | 0.1 | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 11 Public administration (91) | 0 | 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Total | 1133 | 90 | 61 | 43 | 1135.6 | 87.4 | 1136.7 | 86.3 |
| Total gross value added | 728 | 118 | 39 | 57 | 731.4 | 114.6 | 730.3 | 115.7 |
| Total output | 1861 | 208 | 100 | 100 | 1867 | 202 | 1867 | 202 |

28.50 Both these assumptions, the technology assumption and the sales structure assumption, are rather simplistic and in practice a more generalized approach may be used but it is helpful first to examine each of the assumptions in a little more detail.

## Product by product tables

28.51 There are two ways in which a product by product matrix can be derived. These are:
a. The industry technology assumption where each industry has its own specific means of production irrespective of its product mix.
b. The product technology assumption where each product is produced in its own specific way irrespective of the industry where it is produced.
28.52 It is simplest to explain these by example. In the upper part of table 14.12, the construction industry is shown as producing 6 units (out of 208) of manufacturing products. In the lower part of table 14.12, reproduced as table 28.3, the inputs necessary for manufacturing and for construction are shown. These are reproduced in the first two numeric columns in table 28.5. The next two numeric columns express these in percentage form. Thus, for example, one unit of manufacturing requires 0.038 units of agricultural products, 0.102 units of ores and minerals and so on. Construction uses no agricultural products, 0.005 units of ores and minerals and so on.
28.53 In order to create the product by product matrix, it is necessary to deduct the costs associated with the production of 6 units of manufactured goods from the column for construction and add it to the column for manufacturing. On completion of this exercise for all secondary production, the columns will represent products rather than industries.

## Industry technology assumption

28.54 Under the industry technology assumption, the coefficients showing how manufactured products are produced are assumed to depend on the industry they happen to be produced in. Thus to reallocate the 6 units of manufacturing products from the construction industry to a column that will now refer to manufactured products only (ignoring other secondary products for the moment) a set of inputs, derived as 6 times the coefficients for construction is added to the manufacturing column and deducted from the construction column. The results of this are shown in the fifth and sixth numeric columns of table 28.5.

## Product technology assumptions

28.55 Under the product technology assumption, the coefficients showing how manufactured products are produced are those of the manufacturing industry regardless of where they are actually produced. In this case, to reallocate the 6 units of manufacturing products from the construction industry a set of inputs derived as 6 times the coefficients for manufacturing is added to the manufacturing column and deducted from the construction column. The results are
shown in the seventh and eighth numeric columns of table 28.5 .
28.56 It is important to note a problem that arises under this assumption. When the product technology assumption is used, manufactured products produced by the construction industry are assumed to use a small amount of food. However, no agricultural products are actually recorded as being used in the construction industry so deducting these inputs from the recorded entries for construction leads to a negative entry. Negative entries cannot appear under the industry technology assumption. Since negative entries are logically impossible, this is one argument in favour of using the industry assumption rather than the product assumption.

## Industry by industry tables

28.57 Just as there are two ways in which a product by product matrix can be derived, there are two ways in which an industry by industry matrix can be derived. These are:
a. The fixed product sales structure where it is assumed the allocation of demand to users depends on the product and not the industry from where it is sold.
b. The fixed industry sales structure where it is assumed that users always demand the same mix of products from an industry.
28.58 Although a table similar to table 28.5 is not presented for the industry by industry tables, its construction is similar and straightforward but would show the entries across the rows of the use tables rather than down the columns.
28.59 In order to create an industry by industry table, it is necessary to move the use of 6 units of manufactured products from the row for the manufacturing to the row for the construction. On completion of this exercise for all secondary production, the rows will represent industries rather than products.

## Fixed product sales structure

28.60 In this case, to allocate the 6 units of manufactured goods supplied by the construction industry to the row for construction, a proportion of the row for manufacturing is allocated to the construction row using the proportions in the manufacturing row. It follows that such a matrix will not contain negative entries.

## Fixed industry sales structures

28.61 Here the 6 units of manufactured goods supplied by the construction industry are reallocated to the construction row from the manufacturing row using the proportions of the construction row. Such a matrix can contain negative elements.

## The choice of approach to be used

28.62 There are four basic choices open to the input-output compiler.

Table 28.6:Example of a product by product input-output matrix


Table 28.7:Example of an industry by industry input-output matrix

a. A product by product approach using a product technology assumption,
b. A product by product approach using an industry technology assumption,
c. An industry by industry approach assuming a fixed product sales structure,
d. An industry by industry approach assuming a fixed industry sales structure.

Options a and d may result in negative entries; options b and c do not.
28.63 Both product by product and industry by industry tables may be compiled. They serve different analytical functions. For example, to ensure that price indices are strictly consistent, a product by product matrix is to be preferred. For a link to labour market questions, an industry by industry table may be more useful. Although traditionally a lot of interest focused on the product by product tables, this was accompanied in large part by an attention to the underlying technology. Increasingly the economic interaction of different industries has brought more interest in the industry by industry tables.

## Hybrid approaches

28.64 In practice, no single method is used on its own. Knowledge of the type of product or industry in question should dictate whether an industry-based conversion procedure or a product-based one is most appropriate. Some secondary products may be dealt with one way and others another despite the fact that, on occasion, negative values may initially appear.
28.65 The extent of variation between the various approaches will depend on a number of factors, including in particular the extent of secondary production in the supply matrix. In general, the greater the degree of disaggregation and thus the less secondary production to be reallocated, the closer the input-output tables will resemble the supply and use
tables. Indeed some countries prefer to work with very detailed supply and use tables and not produce symmetric tables at all.
28.66 As an illustration of the differences involved, tables 28.6 and 28.7 show the results of converting the supply and use tables in chapter 14 to, first, a product by product matrix using only the industry technology assumption and then an industry by industry matrix using only the product sales structure.

## The database required for the transformation

28.67 The starting point for the production of a symmetric inputoutput table is a pair of supply and a use tables both at basic prices. Even the calculation of a use table in basic prices is one step away from basic statistics and actual observations, reinforcing the fact that the input-output tables are analytical constructs, not a compilation of directly observed phenomena.
28.68 Further, it is advantageous to separate the use table at basic prices into two, one showing those elements relating to domestic output and the other those elements relating to imports. The statistical requirements for such a separation are demanding but the results allow considerable flexibility in the treatment of imports and permit a clear analysis of the impact of demand on supplies from resident producers and on foreign suppliers.
28.69 The exact manner of dealing with imports is a subject of considerable complexity where a number of options are available also. In some economies, some important products will only be imported and so separating these "non-competing" imports from the rest may be of particular interest.
28.70 Another topic that requires careful consideration is the degree of detail that is desirable for product and industry classifications. This may vary depending on the resources available to the statistical office and the sort of use to be made of the results.

Table 28.8:The goods and services account in matrix form


## D. Social accounting matrices

## 1. Expressing the sequence of accounts in matrix form

28.71 The part of the use table relating to the destination of products represents one side of the goods and services account in matrix form. However, it can also be expressed as a series of sub-matrices; one for intermediate consumption, one for final consumption, one for capital formation and one for exports. These sub-elements can be associated with the production account, the use of income account, the capital account and the rest of the world account respectively. Similarly the supply table represents the other side of the goods and services account but can also be written as two sub-matrices, one associated with the production account (output) and one with the rest of the world (imports). By writing the supply table horizontally and the supply table vertically in terms of these submatrices and their associated accounts, table 28.8 emerges. The rows and columns labelled E denote the total economy and those labelled R the rest of the world.
28.72 The attraction of this format is that the total across the set of rows for the goods and services account is equal to the total down the columns for the same account. There is no match for the second set of rows for the production account, but it is not difficult to bring this about. The entries for value added can be inserted in a third set of rows with the entries underneath intermediate consumption. In this way the sum down the columns for the production account is then equal to the rows for the same account. But there is now an unmatched third set of rows containing value added. Since value added ultimately carries forward to the allocation of primary income account, the third set of rows can be so labelled as in table 28.9 .
28.73 If, to match this third set of rows, a third set of columns is inserted between the production account columns and those for the use of income account, property income can be inserted at the intersection of the third set of rows and columns and a fourth set of rows inserted to show the balance of primary income as it appears in the secondary distribution of income account. Proceeding in this way,
successive sets of rows and columns can be introduced until the whole sequence of accounts is covered, as in table 28.10 .
28.74 By including the entries for the rest of the world as well as for the total economy, the balancing items from the balance of payments can be shown as, for instance, the -41 in table 28.9.
28.75 It is also possible to extend table 28.10 to show the incorporation of the balance sheets as in table 28.11. For this, a row above the initial table is introduced to show the opening balance sheet and three rows below it. The first of these shows the entries for the other changes in the volume of assets account, the second relates to the revaluation account and the last is the closing balance sheet. Two adjustments also need to be made to table 28.6. The first concerns the item for the consumption of fixed capital, which is transposed from the row for the capital account and column for the production account and placed in the column for the capital account and row for the production account but with a negative sign. The second is to subdivide the capital account with the first set of rows and columns covering all items in the account but the second set covering the product details for gross capital formation and thus forming part of the asset account for non-financial assets.
28.76 Reading down the columns starting with the opening balance sheet entry for fixed assets, for example, this value plus the value of capital formation, less consumption of fixed capital, plus other changes in the volume of assets plus revaluation items is equal to the value on the closing balance sheet. For financial assets less liabilities the matching identity holds.

## 2. Expanding the matrix

28.77 It is possible to expand and rearrange the rows and columns of the matrix so long as this is done consistently in both dimensions. It is not strictly necessary to adhere to the

Table 28.9:The supply and use table in matrix form

|  | Goods and services account |  | R | Production account | E | R | Use of income accounts | E | R | Capital accounts |  | R | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Goods and services account | Exports Imports | $499$ | 540 | Intermediate consumption | $1883$ |  | Final consumption | $1399$ |  | Gross capital formation | $414$ |  | $\begin{array}{r} 4236 \\ 499 \end{array}$ |
| Production account | Output | 3737 |  |  |  |  |  |  |  |  |  |  | 3737 |
| Primary distribution of income accounts |  |  | -41 | Value added | 1854 |  |  |  |  |  |  |  |  |
| T"'otal |  | 4236 | 499 |  | 3737 |  |  |  |  |  |  |  |  |

order of the sequence of accounts or the degree of detail shown there. The transactions to be included can be expanded or contracted as can the sets of institutional units to be identified.
28.78 The example of transposing consumption of fixed capital from being a positive entry on one side of the account to a negative entry on the other demonstrates how the matrix formulation may be used to enhance the articulation of the asset accounts.
28.79 It is also possible to include alternative classifications of key items. For example a row called "human needs" could be included showing how much food, housing etc was needed for each group of households, based on the functional classification of household consumption. In the column for consumption expenditure, the set of needs can be then cross-classified by product and household group.
28.80 A further expansion of the matrix may be to show the from-whom-to-whom details of such flows as property income and transfers.
28.81 The matrix presentation is very powerful in terms of the flexibility it can encompass, and in displaying the interaction of the accounts in a compact and graphic manner. On the other hand, there are disadvantages to the matrix presentation also.
a. Without explanatory text describing each of the main elements, a reader has to have a very good understanding of the SNA to interpret the numeric entries in the table.
b. Such a table always contains lots of white space which means that it is not an effective way of presenting a large amount of data.

In general, the matrix format is best used to explain the structure of the accounts being presented with individual cells, or a combination of cells, following in a more traditional format.

## 3. Disaggregating households

28.82 Expanding the accounting matrix of the sequence of accounts to incorporate the disaggregation of households is the usual form of a satellite account known as a social accounting matrix (SAM). As such it moves beyond a rigorous accounting structure based on observations to make an allocation of income into household groups possibly based on a household income and expenditure survey. In some cases this is based on a single survey. The problem, as explained in chapter 24 on the household sector, is that income flows in the SNA relate to individuals whether as employees, recipients of property income or transfer recipients while expenditure relates to households.

Mapping individuals to households is necessarily difficult and depends to a greater or lesser extent on a set of assumptions. Any analysis of how government policies will affect households and their consumption depends on making such a mapping.

## 4. A SAM for labour accounts

28.83 One example of where a SAM is useful is in the case of labour accounts, showing the level and composition of employment and unemployment. SAMs have often provided additional information on this issue, via a subdivision of compensation of employees by type of person employed. This subdivision applies to both the use of labour by industry, as shown in the supply and use table, and the supply of labour by socio-economic subgroup, as shown in the allocation of primary income account for households. It implies that the matrix presents not only the supply and use of various products, but also the supply and use of various categories of labour services.
28.84 In order to have a comprehensive picture of the relationship between households and the labour market, the following sets of information are likely to be needed:
a. Various stocks underlying the flows in the SAM, such as size and composition of the population by household group (including the potential labour force) and production capacity by industry;
b. For the self-employed, it may be desirable to have information on the possession of assets (for example, agricultural land, consumer durables) as well as information on financial assets and liabilities;
c. Related non-monetary socio-economic indicators, such as life expectancy, infant mortality, adult literacy, nutrient intake, access to (public) health and education facilities, and housing situation by household group (see Towards a System of Social and Demographic Statistics (United Nations, 1975));
d. Some re-routings such as social transfers in kind by groups of households.
28.85 Comparing labour incomes of all employed persons as shown in the SAM, a decomposition of these incomes into full-time equivalent employment and average wage rates, and the potential labour force by type of person and household group (expressed in "full-time" equivalents), yields detailed information on the composition of unemployment and an aggregate indicator ("full-time equivalent unemployment") which is consistent, both conceptually and numerically, with the other macroeconomic indicators; these can also be derived from the SAM framework.

Table 28.10:The flow accounts in the sequence of accounts in matrix form


Table 28.11:The sequence of accounts including the balance sheets in matrix form


